

CONCOURS D'ENTREE EN 1ère ANNEE – SESSION DE SEPTEMBRE 2018

EPREUVE DE MATHEMATIQUES

Durée 3h00 - Coefficient 4

EXERCISE 1 5 Marks

- 1- Consider the sequence (u_n) defined by: $u_0 = 0$, $u_1 = 1$, for any natural number n ,
- $$u_{n+2} = 5u_{n+1} - 4u_n.$$

Calculate u_2 , u_3 , u_4 of the sequence (u_n) . **0.25 x 3 = 0.75 mark**

2- a) Show that for any natural integer n , $u_{n+1} = 4u_n + 1$. **0.75 mark**

b) Show that for every natural integer n , (u_n) is a natural integer. **0.5 mark**

c) Deduce from the previous questions, that for any natural integer n , the gcd of (u_n) and (u_{n+1}) **1 mark**

3- Let (V_n) be the sequence defined for every integer n by: $V_n = u_n + \frac{1}{3}$

a) Show that (V_n) is a geometric sequence. Find the first term and the common ratio. **0.75 mark**

b) Express V_n and u_n as a function of n . **0.25 x 2 marks**

c) Determine for all integer n , the gcd of $4^n - 1$ and $4^{n+1} - 1$. **0.5 mark**

4- Calculate the difference $(4^{n+1} - 1) - (4^n - 1)$, and obtain the results in part 3.c above.

0.75 mark

EXERCISE 2 5 Marks

Given the matrices $A = \begin{pmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{pmatrix}$ $P = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ and $D = P^{-1} \times A \times P$

1- Calculate P^{-1} and D . **0.5 x 2 marks**

Prove that for every integer n , $A^n = P \times D^n \times P^{-1}$. **1.5 marks**

2- Deduce the expression of A^n as a function of n . **1 mark**

3- Calculate A^2 et A^3 . 0.75 x 2 mark

EXERCISE 3 **5 Marks**

PART A **1.5 Marks**

Let V be the function defined on $]0 ; +\infty[$ by : $V(x) = \frac{2\ln x}{x^2+x}$

1-) Show that for all x greater than or equal to 1, $\frac{\ln x}{x^2} \leq V(x) \leq \frac{\ln x}{x}$. **0.5 mark**

2-) Evaluate $I = \int_1^3 \frac{\ln x}{x^2} dx$ and $J = \int_1^3 \frac{\ln x}{x} dx$; **0.5 mark**

3-) Deduce the interval within which K lies given that $K = \int_1^3 V(x) dx$. **0.5 mark**

PART B **3.5 marks**

f denotes the numerical function of the real variable x defined by: $f(x) = x - \frac{e^x - 1}{e^x + 1}$ and (C) its representative curve in an orthonormal frame. Units on the axes: 2cm.

1- Give the variations of the function f and draw the variations table. **0.75 mark**

2-a-) Show that for all real x , we have: $f(x) - (x - 1) = \frac{2}{e^x + 1} = \frac{2e^{-x}}{1 + e^{-x}}$ **0.25 mark**

b-) Show that the lines (D) and (D') of respective equations: $y = x - 1$ and $y = x + 1$ are asymptotes to the curve (C) of the function f . **0.5 mark**

c-) Draw the lines (D); (D') and the curve (C) of f in the cartesian frame. **0.5 mark**

a-) Show that f admits on \mathbb{R} a reciprocal (f^{-1}) and give its table of variations. **0.5 mark**

b-) Draw the curve (C') of f^{-1} in the same frame as (C). **0.25 mark**

4-Let a be a real number greater than or equal to 1. $A(a)$ denotes the area in cm^2 of the part of the plane bounded by the curve (C), the line (D) and the lines of equations $X = 1$ and $X = a$.

a-) Calculate $A(a)$ and calculate the value of $A(2)$ correct to the nearest one hundredth. **0.5 mark**

b-) Calculate the limit of $A(a)$ as a tends to $+\infty$. **0.25 mark**

EXERCISE 4 5 marks

A bag contains 8 balls of which 5 are black. We draw at the same time 6 balls from the bag.

1- Calculate the probability of obtaining exactly 3 black balls. **0.5 mark**

2- We repeat 10 times consecutively in an identical and independent manner the random withdrawal of 6 balls at the same time from the bag.

Calculate the probability of obtaining exactly 6 times, 3 black balls at the end of the experiment. **1.5 marks**

3- We draw n times consecutively in an identical and independent manner 6 balls from the bag.

Calculate the probability P_n of event E: "obtaining at least one time 3 balls of black colour". **1.5 marks**

4- Find the minimum value of n for which the probability of E is at least 0.95. **1.5 marks**